

VORTICITY ON MHD FREE CONVECTION FLOW OF FLUID WITH HEAT TRANSFER THROUGH POROUS MEDIUM BY AN OSCILLATING POROUS PLATE IN SLIP FLOW REGION

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ABSTRACT

The vorticity of MHD Free convection viscous incompressible fluid past an accelerating infinite vertical plate with heat transfer through porous medium by an oscillating porous plate in slip flow region has been studied. The dimensionless governing equations are solved using ordinary differential equation solution in separation. The vorticity of the flow has been found the different values of various parameters from graphs.

KEYWORDS: MHD, Free Convection, Vertical Plate, Acceleration, Heat Transfer, Vorticity and Porous Medium

INTRODUCTION

The requirements of modern technology have stimulated the interest in fluid flow studies, which involve the interaction of several phenomena. One such study is presented, when a viscous fluid flows over a porous surface, because of its importance in many engineering problems bearing in the field of water in river beds, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purification process by Joseph and Tao [9]. Cunningham and Williams [4] also reported several geophysical applications of flow in porous medium, viz. porous roller and its natural occurrence in the flow of river through porous bank and beds the flow of oil through underground porous rocks.

Free convection effect on flow past a vertical surface studied by Vajnavelu et al [2], Vedhanayagam [3] and others with different boundary conditions. Revankar et al [5] and many workers have studied hydro magnetic natural convection flow past a vertical surface.

Convective heat transfer through porous media has been a subject of great interest for the last three decades. Kim et al [6] and Harris et al [8] have studied the problem of natural convection flow through porous medium past vertical plate. Mishra and Mohapatra [1] have considered the unsteady MHD free convection flow past a vertical porous plate. Gupta and Sharma [7] have been discussed magneto hydrodynamic flow through a porous medium bounded by an oscillating plate in a slip flow regime. Agarwal and Sing [10] give the note on vorticity of MHD flow of viscous fluid through a porous medium bounded by an Oscillating porous plate in slip flow regime.

In this problem, It try to investigate the Vorticity on MHD free convection flow of fluid with heat transfer through porous medium by an oscillating porous plate in slip flow region

FORMULATION OF PROBLEM

Consider the unsteady free convection flow of an incompressible, electrically conducting viscous fluid through

porous medium bounded by an oscillating porous plate in slip flow region with heat transfer . The X' –axis is taken along the plate in the upward direction growing in the direction of motion and Y' –axis is taken normal to the plate. Assume that the fluid has constant properties and the variation in density and mass concentration is considered only in the body force term. A magnetic field of uniform strength B_0 acts normal to the plate. Let u' and v' are the components of velocity along x' -axis and y' axis .As the plate is infinite in length and uniform suction all the physical variable depend only y' and t' except body force. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} + V \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) - \frac{\nu}{K} u' - \frac{\sigma B_0^2}{\rho} u' \quad (1)$$

$$\frac{\partial T'}{\partial t'} + V \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} \quad (2)$$

With boundary conditions

$$\left. \begin{aligned} u' &= u_0 \cos n't' + L \frac{\partial u'}{\partial y'} \quad T' = T'_w \text{ at } y' = 0 \\ u' &= T'_\infty + (T'_w - T'_\infty) \cos n't', \quad T' = T'_\infty \text{ as } y' \rightarrow \infty \end{aligned} \right\} \quad (3)$$

Where g is acceleration due to gravity, T' is the temperature of the fluid, β is the coefficient of thermal expansion, ν is the kinematic viscosity of the fluid, \bar{k} is effective thermal conductivity, ρ is the density of the fluid, k_0 is the permeability, C_p is the specific heat at constant pressure.

Let us introduce the following non dimensional quantities

$$\left. \begin{aligned} y &= \frac{U_0 y'}{\nu}, \quad u = \frac{u'}{U_0}, \quad V = \frac{V'}{U_0}, \quad t = \frac{t' U_0^2}{\nu}, \quad n = \frac{n' \nu}{U_0^2}, \quad K = \frac{U_0^2 K'}{\nu^2}, \quad M = \frac{B_0^2 \nu}{\rho U_0^2} \\ R &= \frac{L U_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad Pr = \frac{\mu C_p}{k}, \quad Gr = \frac{\nu \beta g (T'_w - T'_\infty)}{U_0^3} \end{aligned} \right\} \quad (4)$$

Where Gr is Grash f number, M is magnetic number, Pr is prandtl number, K permeability parameter porous medium and R is rarefaction parameter.

Then using equation (4), equations (1) and (2) with boundary condition (3), reduce to

$$\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - \left(M + \frac{1}{K}\right)u \quad (5)$$

$$\frac{\partial \theta}{\partial t} + V \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (6)$$

With boundary conditions

$$\left. \begin{aligned} u &= \cos nt + R \frac{\partial u}{\partial y}, \quad \theta = 1, \text{ at } y' = 0 \\ u &= \cos nt, \quad \theta = 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (7)$$

SOLUTION OF PROBLEM

Let us assume the solution for velocity u and Temperature θ in the form of

$$u = F(y) \cos(nt - Ay) \quad (8)$$

$$\theta = G(y) \cos(nt - By) \tag{9}$$

Where A and B are constant to be determined. Substituting the value of u and θ from (8) and (9) in (5) and (6), and equating the coefficients of sin and cos terms both sides, we get

$$2AF' + (VA - n)F = 0 \tag{10}$$

$$F'' - VF' - \left(A^2 + M + \frac{1}{K}\right)F = -Gr \cdot G(y) \tag{11}$$

$$G'' - PrG' - B^2G = 0 \tag{12}$$

$$2BG' + (Prn - VPrB)G = 0 \tag{13}$$

With boundary conditions

$$G(0) = 1, (AF(0)\sin nt + F'(0)\cos nt)R = (F(0) - 1)\cos nt, G(\infty) = 0, F(\infty) = 0 \tag{14}$$

By solving equation (10)-(13) using boundary condition (14), we get

$$G = e^{(VB-n)Pr y} \tag{15}$$

$$F = Qe^{\left(A - \sqrt{V^2 + 4\left(A^2 + M + \frac{1}{K}\right)}\right)y} + \left(\frac{-Gr}{S - A^2}\right)e^{(VB-n)Pr y} \tag{16}$$

$$\text{Where } B = \frac{(2n+1)Pr^2V + \sqrt{((2n+1)Pr^2V)^2 - 4(Pr^2V^2 - 1)Pr^2n(n+1)}}{2(Pr^2V^2 - 1)}$$

$$S = ((VB - n)Pr)^2 - VPr(VB - n) - \left(M + \frac{1}{K}\right)$$

$$Q = \frac{Gr(1 - ARtann t) - RGrPr(Vb - n)}{(s - A^2)\left((1 - ARtann t) - R\left(A - \sqrt{4A^2 + V^2 + 4M + \frac{4}{K}}\right)\right)}$$

$$A = \frac{nR}{VR - 2V - 2GrR}$$

Using equation (8) and (16), we get

$$u = \left(Qe^{\left(A - \sqrt{V^2 + 4\left(A^2 + M + \frac{1}{K}\right)}\right)y} + \left(\frac{-Gr}{S - A^2}\right)e^{(VB-n)Pr y} \right) \cos(nt - Ay)$$

From the point of view of its use the vorticity of flow will be

$$\gamma = \left(Q \left(A - \sqrt{V^2 + 4\left(A^2 + M + \frac{1}{K}\right)} \right) e^{\left(A - \sqrt{V^2 + 4\left(A^2 + M + \frac{1}{K}\right)}\right)y} - \frac{Gr(VB-n)Pr}{S - A^2} e^{(VB-n)Pr y} \right) \cos(nt - Ay) + A \left(Qe^{\left(A - \sqrt{V^2 + 4\left(A^2 + M + \frac{1}{K}\right)}\right)y} + \left(\frac{-Gr}{S - A^2}\right)e^{(VB-n)Pr y} \right) \sin(nt - Ay)$$

RESULTS AND DISCUSSIONS

In this paper I have studied the Vorticity on MHD free convection flow of fluid with heat transfer through porous medium by an oscillating porous plate in slip flow region. The effect of the parameters M, V, R, K, Gr, n, and Pr on flow

characteristics have been studied and shown by means of graphs. In order to have physical correlations, we choose suitable values of flow parameters. The graphs of Vorticity of flow are taken w.r.t. y .

The Vorticity of flow profiles are depicted in Figs 1-3. Figure-(1) shows the effect of the parameters V and n on Vorticity at any point of the fluid, when $Pr=2, Gr=2, M=2, R=2, K=2$, and $t=1$. It is noticed that the Vorticity is negative and increases with the increase Suction parameter (V) and Oscillation Parameter (n). It becomes positive when V close to zero and motion becomes irrotational.

Figure-(2) shows the effect of the parameters Pr and Gr on Vorticity at any point of the fluid, when $V=-3, n=2, M=2, R=2, K=2$, and $t=1$. It is noticed that the Vorticity increases with the increase of Prandtl number (Pr), whereas decrease with the increase of Grashof number (Gr),

Figure-(3) shows the effect of the parameters M, R and K on Vorticity at any point of the fluid, when $V=-3, n=2, Gr=2, Pr=2$ and $t=1$. It is noticed that the Vorticity increases with the increase of permeability parameter porous medium (K) and R is rarefaction parameter (R), whereas decrease with the increase of Magnetic parameter (M).

From all the graphs it is found that Vorticity is zero away from the plate and motion of fluid is irrotational.

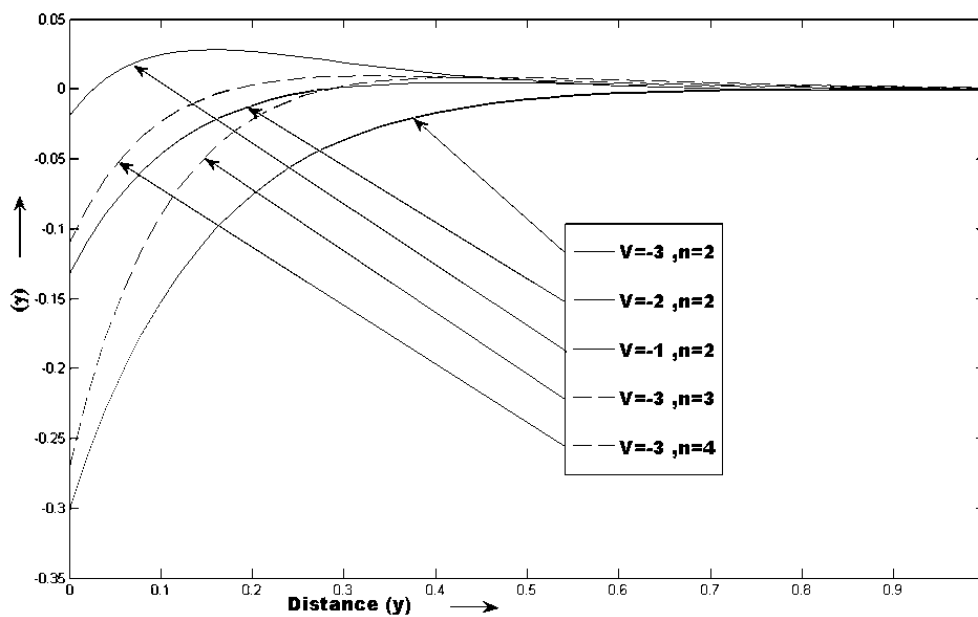


Figure 1: Effect of V and n on Vorticity, when $Pr=2, Gr=2, M=2, R=2, K=2$, and $t=1$

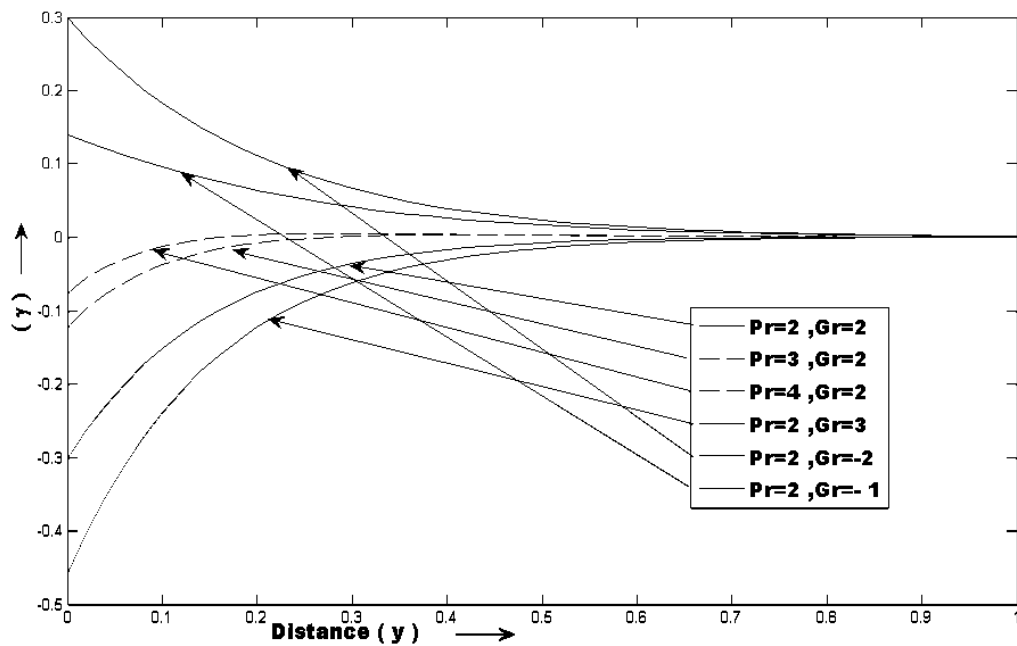


Figure 2: Effect of Pr and Gr on Vorticity, when $V=-3, n=2, M=2, R=2, K=2,$ and $t=1$

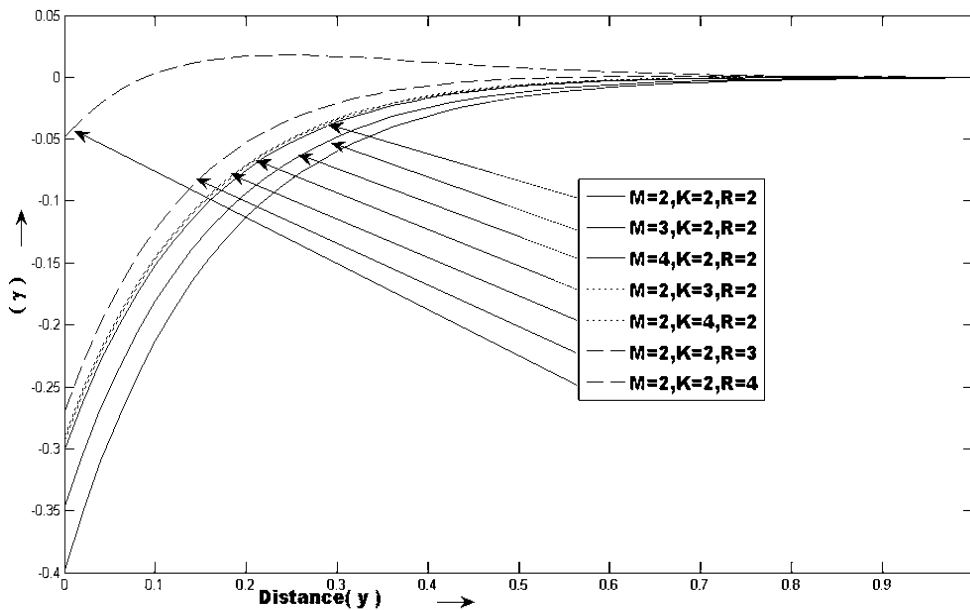


Figure 3: Effect of M, K and R on Vorticity, when $V=-3, Gr=2, Pr=2, R=2$ and $t=1$

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